

The Monge-Ampère equation method in freeform optics design

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Abstract: The Monge-Ampère equation method could be the most advanced point source algorithm of freeform optics design. This paper introduces this method, and outlines two key issues that should be tackled to improve this method.

1. Introduction

In illumination design, freeform surface becomes more and more attractive due to its high degrees of design freedom that can be used to achieve a compact design with an excellent optical performance. The problem of freeform surface illumination design, which requires redirecting the light from a specified light source to produce a desired target illumination, is an inverse problem and a challenging issue. Several kinds of design methods have been proposed to solve this problem, such as the optimization method [1], the simultaneous multiple surface design method [2], the first-order partial differential equation method [3], the method of ellipsoids [4], the method of ray mapping with subsequent surface reconstruction [5] and the Monge-Ampère (MA) equation method [6-8]. Most of these methods rely on the point source assumption, and the MA method that satisfies the integrability condition automatically could be the most advanced point source algorithm [9].

In [8], the problem of freeform surface illumination design is converted into a nonlinear boundary problem for the elliptic MA equation, based on the optimal transport problem, and a numerical technique was developed to solve this nonlinear boundary problem [10]. With this technique, the intercepts of the incident rays on the illumination plane are forced to move automatically to produce the target illumination. In the next section, we will briefly introduce the MA method and the numerical technique. In Section 3, we will outline two key issues that should be addressed to improve the MA method.

2. The elliptic Monge-Ampère equation and numerical technique

Assume the beam shaping system is lossless. So, the power contained in the infinitesimal tube of rays is conserved during the beam shaping process, which is given by

$$E(x, y) |J(\mathbf{T})| = I(\theta, \varphi) \sin \varphi \quad (1)$$

where, $E(x, y)$ is the target irradiance distribution, and $I(\theta, \varphi)$ is the luminous intensity of the point source. Besides, a boundary condition (BC) is required to ensure that the incident rays on the boundary $\partial\Omega$ of domain Ω of the source should be refracted to the boundary ∂R of domain R of the illumination area on the target plane. Then, the single freeform surface illumination design is converted into a nonlinear boundary problem for the elliptic MA equation [8]

$$\begin{cases} A_1 (\rho_{\theta\theta} \rho_{\varphi\varphi} - \rho_{\theta\varphi}^2) + A_2 \rho_{\varphi\varphi} + A_3 \rho_{\theta\theta} + A_4 \rho_{\theta\varphi} + A_5 = 0 \\ BC : \begin{cases} x = x(\theta, \varphi, \rho, \rho_\theta, \rho_\varphi) \\ y = y(\theta, \varphi, \rho, \rho_\theta, \rho_\varphi) \end{cases} : \partial\Omega \rightarrow \partial R \end{cases} \quad (2)$$

where, the coefficients A_i ($i=1, \dots, 5$), are functions of θ , φ , ρ , ρ_θ and ρ_φ . A numerical technique which includes discretization operation, linearization operation and Newton iteration was developed to solve Eq. (2) [10]. With this technique, Eq. (2) is converted into an iterative scheme, which is given by

$$F(\mathbf{X}_k) + F'(\mathbf{X}_k)(\mathbf{X}_{k+1} - \mathbf{X}_k) = 0 \quad (3)$$

where the vector \mathbf{X} represents the variables of Eq. (2), and $F'(\mathbf{X})$ is the Fréchet derivative.

3. To improve the Monge-Ampère method

The MA method has shown its elegance in solving the problem of freeform surface illumination design, while there are two key issues that should be addressed to improve the MA method. The numerical technique employs Newton's method to solve the MA equation. That is, solution of the iterative scheme is determined by the initial design. Thus,

the first issue is to find a proper initial design for iterative scheme shown in Eq. (3). Figure 1(a) gives the irradiance distribution produced by the initial design, and Figure 1(b) gives the illumination pattern obtained from the iterative scheme. It is really hard to imagine that such a high resolution design shown in Fig. 1(b) is obtained from the initial value given in Fig. 1(a). Thus, it is also necessary to figure out how strongly the solution of the iterative scheme is determined by the initial design.

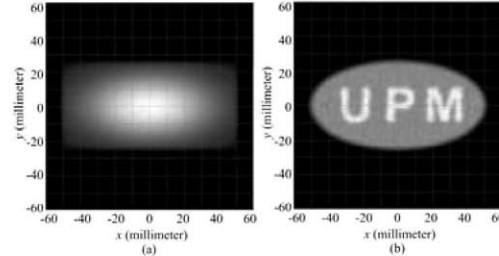


Fig. 1. The illumination patterns (a) produced by the initial design and (b) obtained from the iterative scheme.

As mentioned above, the domain of the source should be discretized first. In the Spherical coordinate system or Cylindrical coordinate system, we can obtain three kinds of grid points after the discretization operation, which are, respectively, the vertex of the freeform surface, the inner grid points and the boundary grid points, as shown in Fig. 2(a). The inner grid points should satisfy the MA equation and the boundary grid points should satisfy the BC. However, the vertex of the freeform surface does not satisfy either the MA equation or the BC. Thus, the second issue is to figure out how to define a proper constraint at the vertex of the freeform surface for the MA equation. The fill operation can avoid complications associated with singularities in the Cylindrical coordinate system shown in Fig. 2 [11], however, it is still very challenging to solve this problem in the Spherical coordinate system.

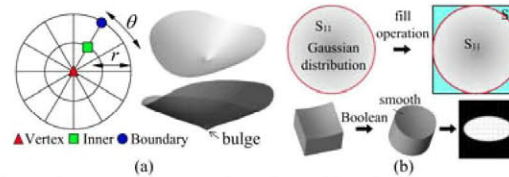


Fig. 2. (a) Discretization operation and bulge at the vertex. (b) Smooth surface with a circular aperture can be obtained by the fill operation.

4. References

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